Math 10B with Professor Stankova
Worksheet, Discussion #24; Thursday, 4/19/2018
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1 Gaussian Elimination

1.1 Concepts

1. In order to solve a system of equations to find the solution or determine if there are zero or infinitely many solutions, use Gaussian elimination on the **augmented matrix**, a matrix formed by appending the answer vector to the original matrix. A system of equations is **consistent** if there is at least one solution and **inconsistent** if there are no solutions.

1.2 Example

2. Use Gaussian elimination on the following augmented matrix. Write the equations these correspond to.

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & -1 & 2 \\ -3 & 0 & -2 & -1 \end{array}\right)$$

1.3 Problems

3. True False As soon as we see a row like $(000\dots0|0)$ during Gaussian elimination, we know that the system will have infinitely many solutions.

4. True False If we see a row like $(000\dots0|0)$ then we know the determinant of the matrix.

5. Come up with an example of a consistent system of equations with 3 equations and 2 variables. Give an example of an inconsistent system of linear equations with 2 equations and 3 variables.

6. Find conditions on a, b such that the following system has no solutions, infinitely many, and a unique solution.

$$\begin{cases} x + ay = 2 \\ 4x + 8y = b \end{cases}$$

7. Use Gaussian elimination to solve the following system of equations:

$$\begin{cases} 2x_1 + x_2 - x_3 = 4 \\ -4x_1 - 2x_2 + 2x_3 = -6 \\ 6x_1 + 3x_2 - 3x_3 = 12 \end{cases}$$

8. Find $\begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 2 & 5 & 2 \end{pmatrix}^{-1}$.

9. Use Gaussian elimination to solve the following system of equations:

$$\begin{cases} x_1 - 2x_2 - 6x_3 = 5 \\ 2x_1 + 4x_2 + 12x_3 = -6 \\ x_1 - 4x_2 - 12x_3 = 9 \end{cases}$$

10. Use Gaussian elimination to solve the following system of equations:

$$\begin{cases} z - 3y = -6 \\ x - 2y - 2z = -14 \\ 4y - x - 3z = 5 \end{cases}$$

1.4 Extra Problems

11. Use Gaussian elimination to solve the following system of equations:

$$\begin{cases} z - 3y = -2 \\ 3y - 4x - 3z = 2 \\ 2z - x - y = -5 \end{cases}$$

12. Use Gaussian elimination to solve the following system of equations:

$$\begin{cases} 2x + 4y - 4z = 0 \\ 5x + y = 6 \\ x - 7y + 8z = -6 \end{cases}$$

13. Find $\begin{pmatrix} 1 & 4 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 0 \end{pmatrix}^{-1}$.

2 Eigenvalues and Eigenvectors

2.1 Concepts

14. An eigenvalue eigenvector pair for a square matrix A is a scalar λ and nonzero vector $\vec{v}$ such that $A\vec{v} = \lambda\vec{v}$. To find this, we write $\lambda\vec{v} = \lambda I\vec{v}$ and bring this to the other side to get $(A - \lambda I)\vec{v} = 0$. Since $\vec{v}$ is nonzero, this means that $(A - \lambda I)\vec{w} = 0$ has at least two solutions (since the trivial solution is a solution), and hence there must be an infinite number of solutions and $\det(A - \lambda I) = 0$.

    So to find the eigenvalues, we solve $\det(A - \lambda I) = 0$. For a particular eigenvalue, to find the associated eigenvector, we have to use Gaussian elimination on $A - \lambda I$ to get the general solution.

2.2 Example

15. Find the eigenvalue and associated eigenvectors of $A = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$.

2.3 Problems

16. True False Associated to every eigenvalue is an eigenvector and vice versa
17. True False If 2 is an eigenvalue for A, then 4 is an eigenvalue for A^2.
18. True False If $\det(A) = 0$, then 0 has to be an eigenvalue of A.
19. True False If 2 is an eigenvalue of A and 3 is an eigenvalue of B, then $2 \cdot 3 = 6$ is an eigenvalue of AB.
20. True False For each eigenvalue, there is only one choice of eigenvector.
21. Find the eigenvalues and eigenvectors of $\begin{pmatrix} 1 & 3 \\ 9 & -5 \end{pmatrix}$.
22. Find the eigenvalues and eigenvectors of $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$.
23. Find the eigenvalues of $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
24. Construct a matrix with eigenvalues 3 and -1.
25. Find the eigenvalues of $\begin{pmatrix} 2 & 4 & 4 \\ -1 & 0 & -1 \\ 1 & 0 & 1 \end{pmatrix}$.